

An introduction to geostatistical analysis of spatio-temporal data with R

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1. Handling and importing large spatio-temporal data using structured objects; projection coordinate systems for geolocated data (*Eric Parent*).
2. **Visualizing data according to their temporal, spatial or spatio-temporal structures (Edith Gabriel).**
3. Statistical inference for spatio-temporal models: method of moments; maximum likelihood, pairwise composite likelihoods (*Thomas Opitz*).
4. Prediction and validation (*Liliane Bel*).

Plotting spatio-temporal data

- *animation* (or a movie): evolution of the spatial data through time.
- *Space (1-D)/time plots*: data in a space-time cross section.
- *Multi-panel spatial maps*: spatial maps for given times or aggregates over time.
- *Time series plots*: time series associated with different stations.

Plotting spatio-temporal data

Spatial location	Time	Number of variables	Vizualisation Package	Function	Dependencies Package	Function
1	n_t	1	tseries ggplot2	irts autoplot	stats	acf
1	n_t	N	tseries ggplot2	irts autoplot	stats	acf
n_s	1	1	sp plotKML	spplot plotKML	gstat gstat	variogram plot.gstatVariogram
n_s	1	N	sp	spplot	gstat gstat	variogram plot.gstatVariogram
n_s	n_t	1	tseries ggplot2 plotKML spacetime	irts autoplot plotKML stplot	stats gstat gstat	acf variogram plot.StVariogram
n_s	n_t	N	sp	spplot		

Spatio-temporal dependence structures

Variogram of a Gaussian process Z ,

$$\gamma((s_1, t_1), (s_2, t_2)) = \frac{1}{2} \mathbb{E} [Z(s_1, t_1) - Z(s_2, t_2)]^2.$$

Related to the covariance function under the assumption of stationarity:

$$\gamma(s_1 - s_2, t_1 - t_2) = \sigma^2 - C(s_1 - s_2, t_1 - t_2).$$

Recall spatial empirical variogram:

$$\hat{\gamma}(h) = \frac{1}{2|N(h)|} \sum_{(i,j) \in N(h)} (Z(s_i) - Z(s_j))^2,$$

where $|N(h)|$ is the number of pairs in the set $N(h) = \{(s_i, s_j) : \text{dist}(s_i, s_j) \leq h\}$.

Spatio-temporal dependence structures

Spatio-temporal variogram:

1. Regular in time: space-time lags are multiples of δ_t .

$$\hat{\gamma}(h, k) = \frac{1}{2|N(h)|} \sum_{(i,j) \in N(h)} (Z(s_i, t) - Z(s_j, t + k\delta_t))^2,$$

where $|N(h)|$ is the number of pairs in the set $N(h) = \{(s_i, s_j) : \text{dist}(s_i, s_j) \simeq h\}$; k time lags apart.

2. Irregular in time: space-time lags are grouped into space-time bins.

$$\hat{\gamma}(h, u) = \frac{1}{2|N(h)|} \sum_{(i,j) \in N(h)} (Z(s_i, t_i) - Z(s_j, t_j))^2,$$

where $|N(h)|$ is the number of pairs in the set $N(h) = \{(s_i, s_j) : \text{dist}(s_i, s_j) \simeq h\}; |t_j - t_i| \simeq u$.

Spatio-temporal dependence structures

Important features of spatio-temporal variograms are:

- Parametric form of the variogram: in space and time
- Regularity of the variogram: nugget effect and smoothness of variogram at the origin can be different in space and in time
- Range in space and time
- **Separability** between space and time

Models for $\gamma(h, u)$ will be presented later