# Joint Classification and Reconstruction of Irregularly Sampled Satellite Multivariate Image Times Series

## Journée RESSTE: Statistiques spatio-temporelles pour les données issues de la télédétection


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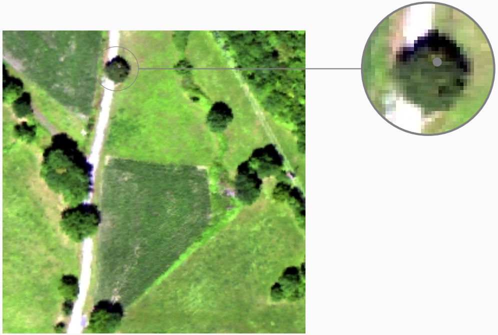
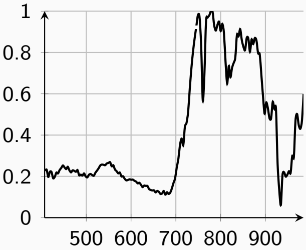
# Outline

1. Satellite Image Time Series for Earth Observation
2. Mixture of Multivariate Gaussian Processes
3. Experimental results
4. Conclusions et prospects

# Satellite Image Time Series for Earth Observation

A remote sensing image is a sampling of a spatial, temporal and spectral process.





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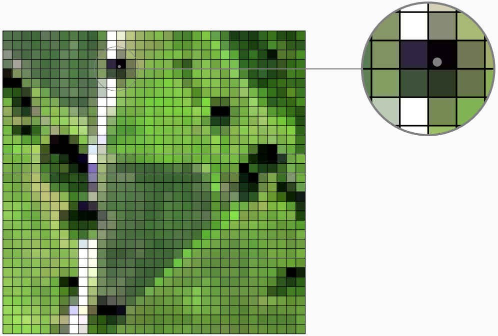

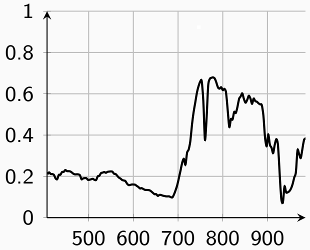

# Remote Sensing Images

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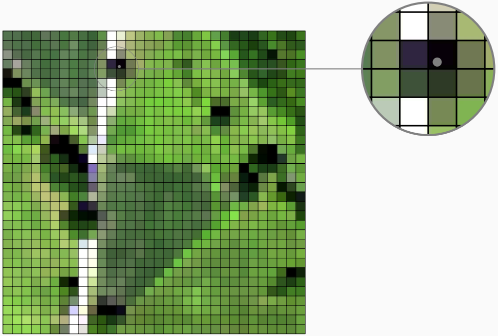

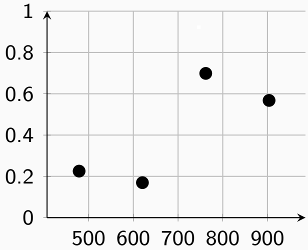

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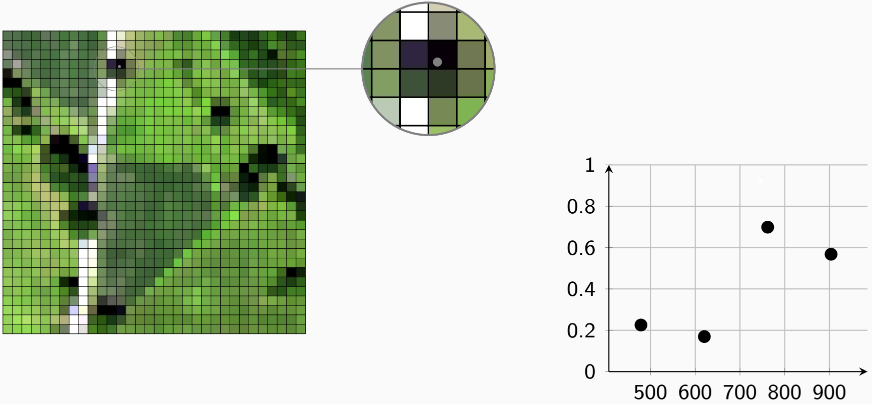


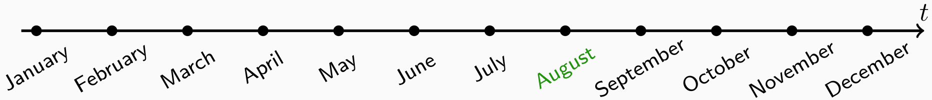

# High Spatial Resolution Satellite Image Time Series

**Sentinel 2**

- High spatial resolution (10m)
- 13 spectral bands
- Revisit every 5 days
- Free and open source data

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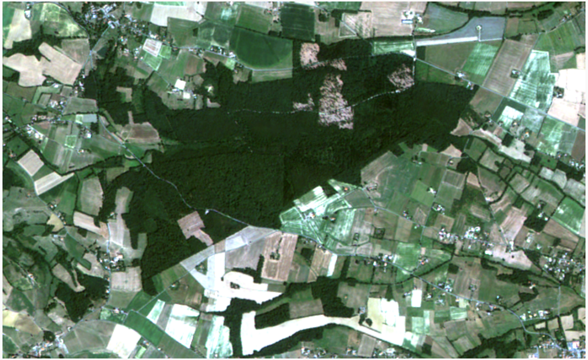

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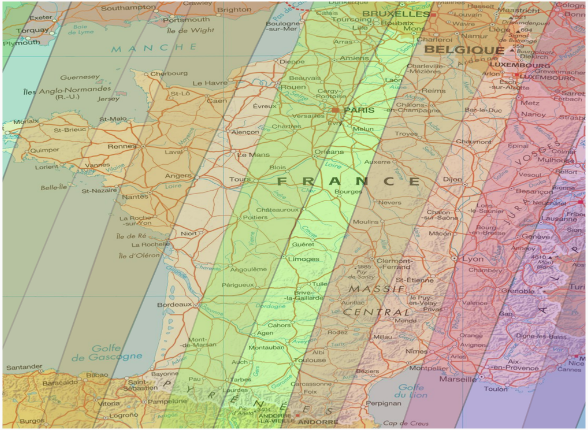
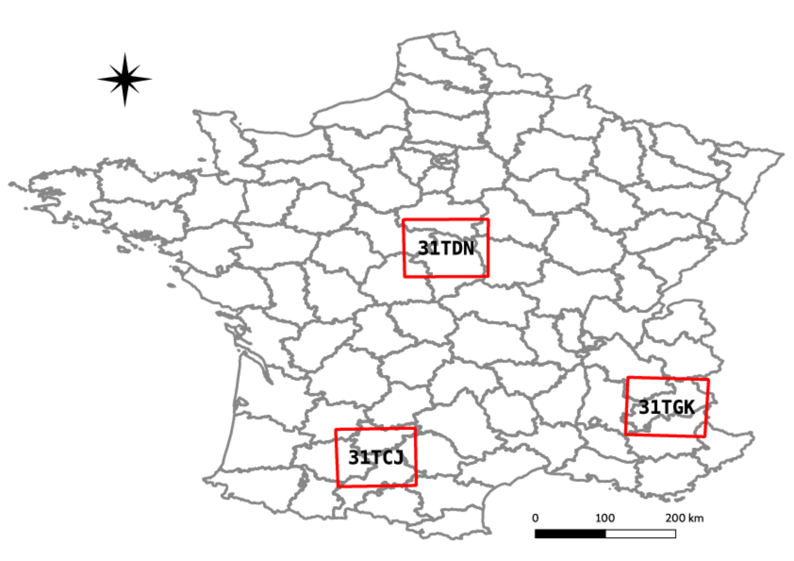


# Irregularly sampled SITS



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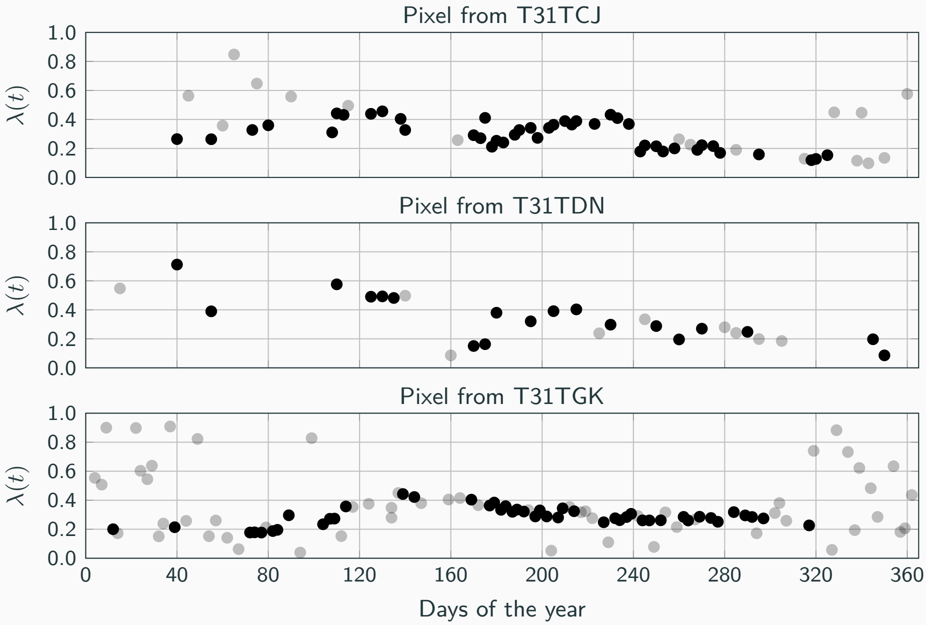
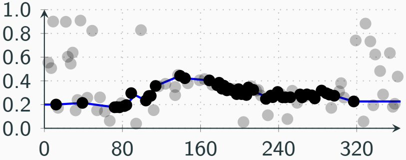


## Pixel-wise classification of SITS

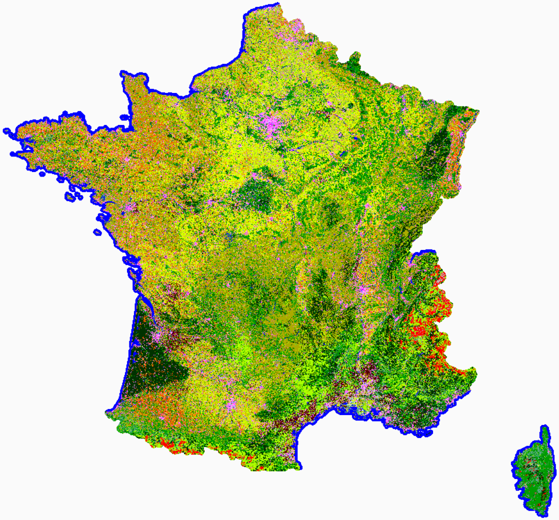
- Temporal resampling




- Input feature

$$\mathbf{x} = \big[\lambda_1(t_1), \ldots, \lambda_1(t_T), \ldots, \lambda_{10}(t_T), \mathsf{si}_1(t_1), \ldots, \mathsf{si}_p(t_T) \ldots\big]$$

- Each sample is associated with class membership $Z$, $Z \in \{1, \ldots, C\}$



# Objectives & Contributions

### Limitation of (some) current approaches

Two-step approaches might not be optimal w.r.t. final objective

- Resampling: reconstruction error
- Classification: likelihood, hindge loss

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- Classification of raw and irregularly sampled SITS
- Scale well w.r.t. the number of samples

### Contribution

- Multivariate Gaussian Processes
- Bayesian classification and imputation

# Mixture of Multivariate Gaussian Processes

**This section is based on**

**Alexandre Constantin, Mathieu Fauvel, Stéphane Girard. Mixture of multivariate gaussian processes for classification of irregularly sampled satellite image time-series. [CFG21b]**

# Mixture of Multivariate Gaussian Processes

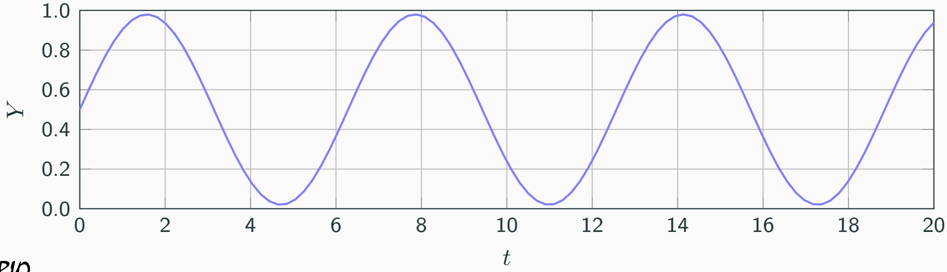
## Definition

### Definition

A Gaussian process (GP) is a stochastic process such that any finite-dimensional marginal follows a multivariate Gaussian distribution [RW05].

$$Y \sim \mathcal{GP}(m, K),\ Y(t) \in \mathbb{R}$$

$$\begin{aligned} m(t) &= \mathbb{E}[Y(t)] \\ K(t,t') &= \mathbb{E}[(Y(t) - m(t))(Y(t') - m(t'))] \end{aligned}$$



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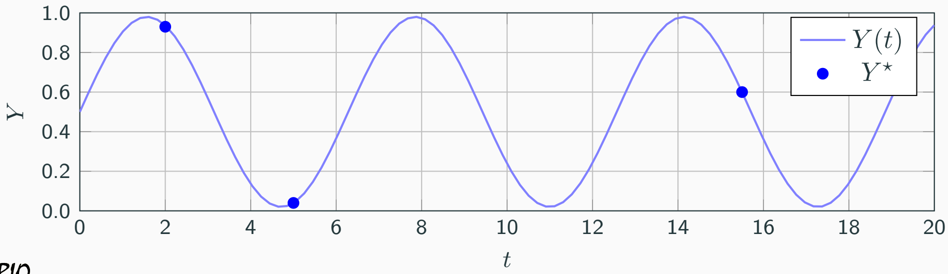
\end{aligned}
$$

For $Y$ observed at $t_1, t_2, t_3$:

$$
\begin{bmatrix} Y^\star(t_1) \\ Y^\star(t_2) \\ Y^\star(t_3) \end{bmatrix} \sim \mathcal{N}_3 \left( \begin{bmatrix} m(t_1) \\ m(t_2) \\ m(t_3) \end{bmatrix}, \begin{bmatrix} K(t_1, t_1) & \dots & K(t_1, t_3) \\ \vdots & \ddots & \vdots \\ K(t_3, t_1) & \dots & K(t_3, t_3) \end{bmatrix} \right)
$$



# Matrix-Variate Gaussian Distribution

**Definition**

The matrix-variate normal distribution $\mathcal{MN}_{p,q}$ [Daw81] is defined for all $p \times q$ random matrices $\mathbf{Y}^\star$ as:

$$\mathbf{Y}^\star \sim \mathcal{MN}_{p,q}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Lambda}) \text{ if and only if } \operatorname{vec}(\mathbf{Y}^\star) \sim \mathcal{N}_{pq}(\operatorname{vec}(\mathbf{M}), \mathbf{\Sigma} \otimes \mathbf{\Lambda}),$$

where $\mathbf{M}$ is a $p \times q$ matrix, $\mathbf{\Sigma}$ and $\mathbf{\Lambda}$ are symmetric positive definite matrices of size $q \times q$ and $p \times p$ respectively.

- If $\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ then $\operatorname{vec}(\mathbf{M}) = [m_{11}, m_{21}, m_{12}, m_{22}]^\top$
- Konecker product: $\mathbf{\Sigma} \otimes \mathbf{\Lambda} = \begin{bmatrix} \sigma_{11}\mathbf{\Lambda} & \cdots & \sigma_{1q}\mathbf{\Lambda} \\ \vdots & \ddots & \vdots \\ \sigma_{q1}\mathbf{\Lambda} & \cdots & \sigma_{qq}\mathbf{\Lambda} \end{bmatrix}$
- PDF: $p(\mathbf{Y}^\star) = (2\pi)^{-pq/2} \left|\mathbf{\Sigma}\right|^{-p/2} \left|\mathbf{\Lambda}\right|^{-q/2} \exp\left(-\frac{1}{2}\operatorname{tr}\left[\mathbf{\Lambda}^{-1}(\mathbf{Y}^\star - \mathbf{M})\mathbf{\Sigma}^{-1}(\mathbf{Y}^\star - \mathbf{M})^\top\right]\right)$

# Mixture of Multivariate Gaussian Processes

## Mixture Model

# Multivariate Gaussian Processes (MGP)

## Definition

We define a Multivariate Gaussian Processes/ (MGP)[a], conditionally to $Z = c$, as

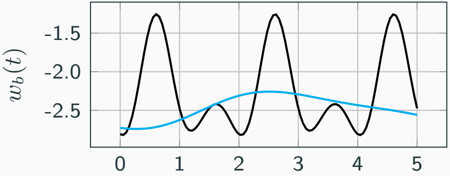
$$\mathbf{Y}(t) = \mathbf{A}_c \mathbf{W}_c(t) + \mathbf{m}_c(t)$$

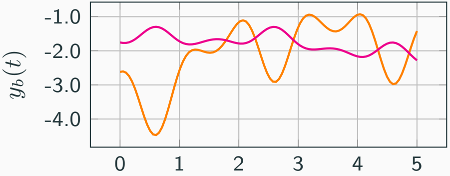
with $\mathbf{Y}(t) \in \mathbb{R}^p$, $\mathbf{W}_c(t) \in \mathbb{R}^p$, $\mathbf{m}(t) \in \mathbb{R}^p$, $\mathbf{A}_c \in \mathbb{R}^{p \times p}$ and

$$\begin{cases} W_{cb} |\ Z = c \sim \mathcal{GP}(0, K_c), \forall\ b \in \{1, \ldots, p\}, \\ W_{cb} \perp W_{cb'}, \forall\ b, b' \in \{1, \ldots, p\}^2. \end{cases}$$

We write $\mathbf{Y}|Z = c \sim \mathcal{MGP}_p(\mathbf{m}_c, K_c, \mathbf{A}_c)$

[a]Special case of *Linear Model of Coregionalization* [Goo97]



# Mixture of MGP

## Definition

We define the *Mixture of MGP* (M2GP) as

$$\mathbf{Y} \sim \sum_{c=1}^{C} \mathbb{P}(Z = c)\mathcal{MGP}_p(\mathbf{m}_c, K_c, \mathbf{A}_c)$$

## Proposition

*If* $\mathbf{Y} \sim$ *M2GP then* $\mathbf{Y}^\star = [\mathbf{Y}(t_1), \ldots, \mathbf{Y}(t_q)]$ *is a* $p \times q$ *random matrix such as:*

$$\mathbf{Y}^\star |\ Z = c \sim \mathcal{MN}_{p,q}(\mathbf{M}_c, \mathbf{\Sigma}_c, \mathbf{A}_c\mathbf{A}_c^\top),$$

*with*

- $\mathbf{M}_c = (\mathbf{m}_c(t_1), \ldots, \mathbf{m}_c(t_q))$,
- $\mathbf{\Sigma}_c$ *is the covariance matrix defined by* $\mathbf{\Sigma}_{c_{l,l'}} = K_c(t_l, t_{l'})$ *for all* $(l, l') \in \{1, \ldots, q\}^2$.

*Equivalently:* $\mathrm{vec}(\mathbf{Y}^\star)|Z = c \sim \mathcal{N}_{pq}(\mathrm{vec}(\mathbf{M}_c), \mathbf{\Sigma}_c \otimes \mathbf{A}_c\mathbf{A}_c^\top)$.

# Model parameters

## Parametric mean

- $\mathbf{m}_c$ is modeled by a weighted sum of $J$ basis functions $\{\varphi_j\}_{j=1}^J$

$$\mathbf{m}_{c,b}(t) = \sum_{j=1}^{J} \alpha_{c,b,j}\varphi_j(t)$$

- Let $\mathbf{B}$ be the design matrix of size $J \times q$ and $\boldsymbol{\alpha}_c = [\boldsymbol{\alpha}_{c,1}, \ldots, \boldsymbol{\alpha}_{c,p}]^\top$ a matrix of size $p \times J$ then

$$\mathbf{M}_c = \boldsymbol{\alpha}_c\mathbf{B}$$

## Parametric kernel

K is chosen in a set of parametric kernel indexed by $\boldsymbol{\theta}$:

$$\boldsymbol{\Sigma}_c = \boldsymbol{\Sigma}(\boldsymbol{\theta}_c)$$

## Reparametrization

$$\mathbf{Y}^{i,\star}|Z^i = c \sim \mathcal{MN}_{p,q_i}\left(\boldsymbol{\alpha}_c\mathbf{B}^i, \boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c), \mathbf{A}_c\mathbf{A}_c^\top\right)$$

# Mixture of Multivariate Gaussian Processes

## Inference

# Maximum Likelihood Estimation

- Let $\mathcal{S} = \{\mathbf{Y}^{i,\star}, Z^i\}_{i=1}^n$ a set of $n$ *i.i.d.* samples.
- Optimal $\left(\boldsymbol{\alpha}_c, \boldsymbol{\theta}_c, \mathbf{A}_c\mathbf{A}_c^\top\right)$ are found by minimizing the *nll*

$$
\begin{aligned}
\ell\left(\boldsymbol{\alpha}_c, \boldsymbol{\theta}_c, \mathbf{A}_c\mathbf{A}_c^\top\right) = & p \sum_{i|Z^i=c} \log|\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c)| + n_c \log|\mathbf{A}_c\mathbf{A}_c^\top| \\
& + \operatorname{tr}\left[\sum_{i|Z^i=c} \left(\mathbf{Y}^{i,\star} - \boldsymbol{\alpha}_c\mathbf{B}^i\right)\left\{\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c)\right\}^{-1}\left(\mathbf{Y}^{i,\star} - \boldsymbol{\alpha}_c\mathbf{B}^i\right)^\top\left\{\mathbf{A}_c\mathbf{A}_c^\top\right\}^{-1}\right] \\
& + \kappa
\end{aligned}
$$

- Solve $C$ independent minimization problems

# Iterative algorithm

Iterate the following steps until convergence:

1. $\boldsymbol{\alpha}_c$ update:

$$\boldsymbol{\alpha}_c^{(k+1)} = \left[\sum_{i|Z^i=c} \mathbf{Y}^{i,\star}\left\{\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c^{(k)})\right\}^{-1}\mathbf{B}^{i^\top}\right]\left[\sum_{i|Z^i=c} \mathbf{B}^{i}\left\{\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c^{(k)})\right\}^{-1}\mathbf{B}^{i^\top}\right]^{-1}$$

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2. $\mathbf{A}_c\mathbf{A}_c^\top$ update:

$$\left(\mathbf{A}_c\mathbf{A}_c^\top\right)^{(k+1)} = \frac{1}{n_c}\sum_{i|Z^i=c}\left(\mathbf{Y}^{i,\star} - \boldsymbol{\alpha}_c^{(k+1)}\mathbf{B}^i\right)\left\{\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c^{(k)})\right\}^{-1}\left(\mathbf{Y}^{i,\star} - \boldsymbol{\alpha}_c^{(k+1)}\mathbf{B}^i\right)^\top$$

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3. $\boldsymbol{\theta}_c$ update:

$$\boldsymbol{\theta}_c^{(k+1)} = \boldsymbol{\theta}_c^{(k)} - \gamma_k \nabla_{\boldsymbol{\theta}_c}\ell_c$$

- $\nabla_{\boldsymbol{\theta}_c}\ell_c = \sum_{i|Z^i=c} \mathrm{tr}\left[\left(p\{\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c^{(k)})\}^{-1} - \beta_c^{i\top}(\mathbf{A}_c\mathbf{A}_c^\top)^{(k+1)}\beta_c^i\right)\frac{\partial\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c^{(k)})}{\partial\boldsymbol{\theta}_c}\right]$
- $\beta_c^i = \left(\mathbf{Y}^{i,\star} - \boldsymbol{\alpha}_c^{(k+1)}\mathbf{B}^i\right)\{\boldsymbol{\Sigma}^i(\boldsymbol{\theta}_c^k)\}^{-1}$

# Mixture of Multivariate Gaussian Processes

## Classification and Imputation

# Classification

- New observation: $p \times q$ random matrix $\tilde{\mathbf{Y}}^{\star} = \left[\tilde{\mathbf{Y}}(\tilde{t}_1), \ldots, \tilde{\mathbf{Y}}(\tilde{t}_q)\right]$
- Posterior probability:

$$\mathbb{P}(\tilde{Z} = c|\tilde{\mathbf{Y}}^{\star}) \propto \pi_c p(\tilde{\mathbf{Y}}^{\star}|\tilde{Z} = c)$$

- MAP with MLE parameters

$$\tilde{c} = \arg\min_{c} \Bigg\{ p \log|\tilde{\boldsymbol{\Sigma}}(\hat{\boldsymbol{\theta}}_c)| + q \log|\widehat{\mathbf{A}_c\mathbf{A}_c^{\top}}| - 2\log(n_c/n) \\ + \operatorname{tr}\left[\left\{\widehat{\mathbf{A}_c\mathbf{A}_c^{\top}}\right\}^{-1}(\tilde{\mathbf{Y}}^{\star} - \hat{\boldsymbol{\alpha}}_c\tilde{\mathbf{B}})\left\{\tilde{\boldsymbol{\Sigma}}(\hat{\boldsymbol{\theta}}_c)\right\}^{-1}(\tilde{\mathbf{Y}}^{\star} - \hat{\boldsymbol{\alpha}}_c\tilde{\mathbf{B}})^{\top}\right] \Bigg\}$$

# Imputation of missing values

Given a sample $\mathbf{Y}^{i,\star}$ observed at $\{t_1, \ldots, t_{q_i}\} = T_i$ times we want to **impute value** at a unobserved time $t^*$

- Conditionally to $Z = c$ and $\mathbf{Y}^{i,\star}$

$$\mathbf{Y}(t^*) \sim \mathcal{N}_p\Big(\boldsymbol{\mu}_c(t^*, \mathbf{Y}^{i,\star}), \boldsymbol{\Lambda}_c(t^*, T^i)\Big)$$

  with
  - $\boldsymbol{\mu}_c(t^*, \mathbf{Y}^{i,\star}) = \hat{\boldsymbol{\alpha}}_c \mathbf{b}^* + \big(\mathbf{Y}^{i,\star} - \hat{\boldsymbol{\alpha}}_c \mathbf{B}^i\big)\big\{\boldsymbol{\Sigma}^i(\hat{\boldsymbol{\theta}}_c)\big\}^{-1}\mathbf{k}(t^*, T_i|\hat{\boldsymbol{\theta}}_c)$
  - $\boldsymbol{\Lambda}_c(t^*, T^i) = \Big[K(t^*, t^*|\hat{\boldsymbol{\theta}}_c) - \mathbf{k}(t^*, T_i|\hat{\boldsymbol{\theta}}_c)^\top\big\{\boldsymbol{\Sigma}^i(\hat{\boldsymbol{\theta}}_c)\big\}^{-1}\mathbf{k}(t^*, T_i|\hat{\boldsymbol{\theta}}_c)\Big] \otimes \widehat{\mathbf{A}_c\mathbf{A}_c^\top}$
- Imputation using MAP:

$$\tilde{\mathbf{Y}}^i(t^*) = \boldsymbol{\mu}_c(t^*, \mathbf{Y}^{i,\star})$$

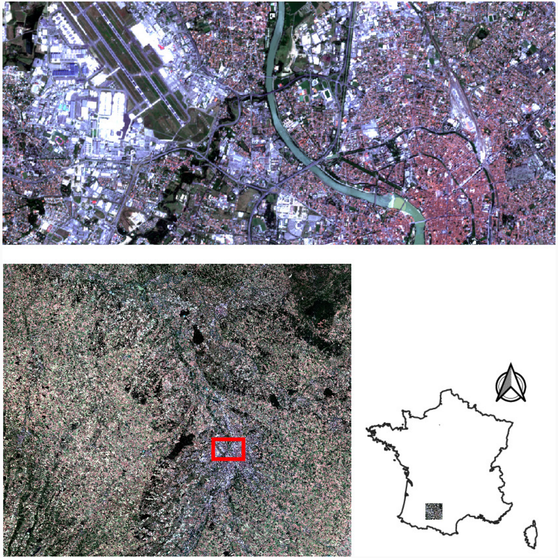
- When class is unknown

$$\mathbf{Y}(t^*) \sim \sum_{c=1}^{C} \mathbb{P}(Z = c|\mathbf{Y}^{i,*})\mathcal{N}_p\Big(\boldsymbol{\mu}_c(t^*, \mathbf{Y}^{i,\star}), \boldsymbol{\Lambda}_c(t^*, T^i)\Big)$$

# Experimental results

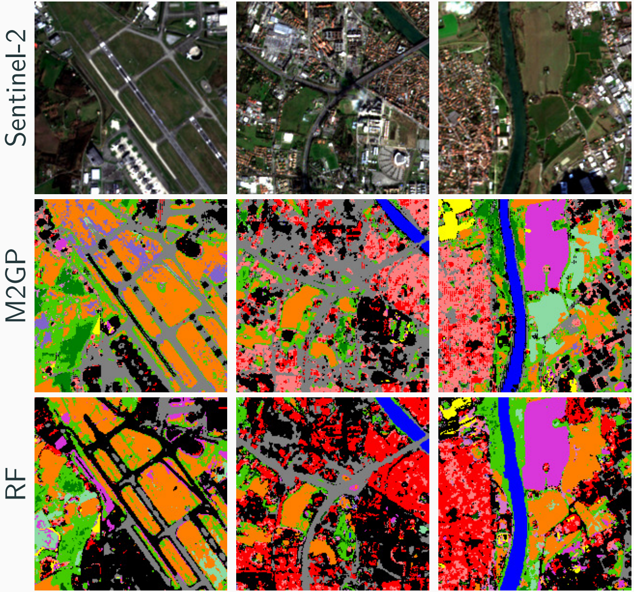
# Sentinel-2 SITS

| Class | $n_c$ |
|---|---|
| Summer crops | 40,000 |
| Winter crops | 30,000 |
| Broad-leaved forest | 10,000 |
| Continuous urban fabric | 10,000 |
| Discontinuous urban fabric | 10,000 |
| Industrial or commercial units | 10,000 |
| Meadow | 10,000 |
| Orchards | 10,000 |
| Road surfaces | 10,000 |
| Vines | 10,000 |
| Water bodies | 10,000 |
| Woody moorlands | 9,972 |
| Coniferous forest | 9,957 |
| Natural grasslands | 9,939 |
| Total | 189,868 |

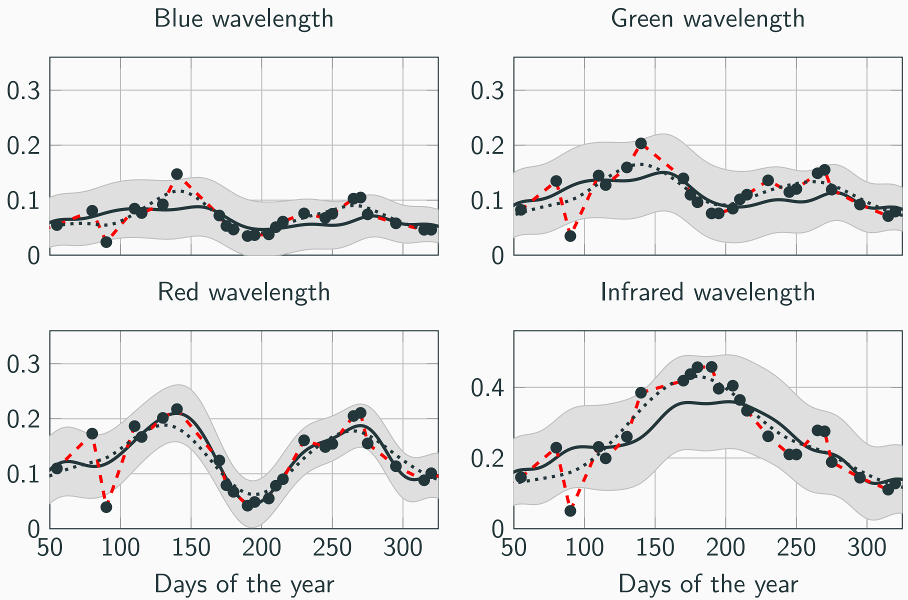


# Classification results

| | Average F1 | std F1 |
|---|---|---|
| MIMGP | 57.4 | 1.04 |
| M2GP | 70.1 | 0.43 |
| QDA | 70.5 | 0.75 |
| lin-SVM | 75.2 | 1.11 |
| RF | 78.2 | 1.17 |

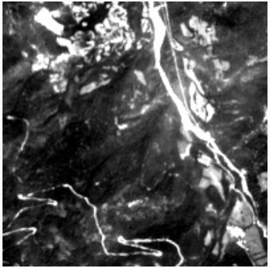


# Reconstruction of missing values 1/2

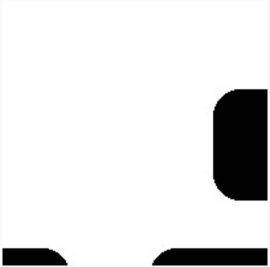


# Reconstruction of missing values 2/2

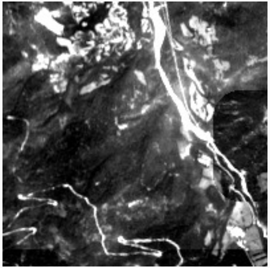
Original



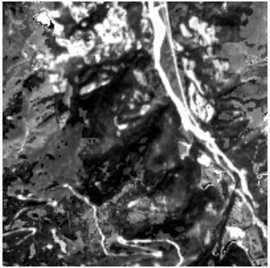
Mask



Whittaker



MIMGP



# Conclusions et prospects

# Conclusions

- ✓ Mixture Multivariate of Gaussian process
- ✓ Bayesian classification and imputation
- ✓ Scale well w.r.t. the number samples
- ✓ Imputation
- ✗ Classification accuracy[1]

[1] Additional results in A. Constantin, M. Fauvel and S. Girard, "Joint Supervised Classification and Reconstruction of Irregularly Sampled Satellite Image Times Series," in *IEEE Transactions on Geoscience and Remote Sensing*, [CFG21a]

# Prospects

- Non-Gaussian Processes
- Unlock spatial stationarity
- Remove the constant sampling assumption for the spectral domain

# References


- Constantin, Alexandre, Mathieu Fauvel, and Stéphane Girard. "Joint Supervised Classification and Reconstruction of Irregularly Sampled Satellite Image Times Series". In: *IEEE Transactions on Geoscience and Remote Sensing* (2021), pp. 1–13. DOI: 10.1109/TGRS.2021.3076667.
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https://gitlab.cesbio.omp.eu/fauvelm/resste_spatio_stat_2022

# Questions / Comments ?